Physics 212: Statistical mechanics II

Problem set 2: due on Friday 3/7/14, 5 pm, envelope outside 523 Birge (new location!)

Reading: Lecture notes up to 12; Kardar 9.1; Kardar chapter 6 (lattice systems)

1. A quick numerical problem: Suppose that a sphere of radius 1 mm moves at 1 mm/sec through a monatomic He gas at room temperature and atmospheric pressure (1 atm = $10^5$ Pa). Estimate the viscosity coefficient of the gas $\eta \approx \sqrt{mkT/a^2}$, taking $a$, the particle size, to be 1 angstrom. What is the viscous drag force on the sphere from Stokes’s law? Estimate the Reynolds number.


3. Write a Langevin equation for 2D Brownian motion of a particle with mass $m$ and charge $e$ in a perpendicular magnetic field $B = B\hat{z}$. Suppose that the drag force is $-m\gamma \mathbf{v}$. If the random force $F(t)$ is white noise, what is the power spectrum of the velocity $\mathbf{v}(t)$? (Can you see two different time scales in this answer?)

4. Suppose the configurations of a polymer of length $N$ are modeled as an $N$-step random walk in $d = 1$: each step has length $L$ and one end of the polymer is fixed at the origin. Derive an approximate Gaussian distribution of the location of the other end of the polymer in dimensionality $d = 1$, assuming large $N$, by calculating the increase of the variance of the polymer end-to-end distance with step:

$$P(x, N) \approx \frac{1}{\sqrt{2\pi NL^2}} e^{-x^2/2NL^2}.$$ 

(You can do this by computing the variance $\langle x^2 \rangle$ of the polymer directly, and showing that it agrees with the above Gaussian; you do not need to prove that a Gaussian is a good fit, unless you want to.)

At a nonzero temperature $T$, what is the entropic force (direction and magnitude) on the polymer when its two ends are at two fixed points a distance $R$ apart, assuming the Gaussian distribution above? Hint: how does the entropic part of free energy change with $R$?

5. Additive Markovian processes: take $L = 1$ in the Gaussian distribution of problem 4, so that the above random walk takes place on the integers. Estimate the probability that after $10^4$ steps the end of the polymer winds up more than 200 steps to the right of the origin by deriving and applying the following asymptotic expansion for the Gaussian distribution:

$$\frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-x^2/2} dx \sim \frac{1}{x\sqrt{2\pi}} e^{-x^2/2}, \quad x \gg 1.$$ 

6. Kardar problem 6.2 (solution at end)